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ON THE VIBRATION OF A ROTATING BLADE ON A TORSIONALLY FLEXIBLE SHAFT

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1. INTRODUCTION

The problem of vibration in rotating machinery has attracted the attention and the efforts of many researchers and maintenance engineers due to economical, strategical and safety consideration. In this regard many accomplishments have been achieved in the understanding of the dynamics and control of the vibration of rotor-bearing system. However, the dynamics of basic rotating components such as rotating blades still need more theoretical and experimental studies. Moreover, the dynamic interaction between the rotating blades and the rotor system dynamics is not yet understood and more theoretical studies are required. One of these studies should address the interaction between blade bending vibration and supporting shaft torsional vibrations.

Srinivasan [1] reported a survey on the vibrations of bladed-disk assemblies. In this survey, he highlighted the importance of modelling the disk–blade vibrations and classified these vibrations into two main categories; namely, structure-induced vibrations and aeroelastic-induced vibrations. The survey was mainly concerned with the structural-induced vibrations and their modeling. Lowely and Khader [2] studied the effect of flexural shaft flexibility on the dynamics of bladed-disk assemblies. Crawely and Mokadam [3] reported analytical and experimental results on the coupled blade–disk–shaft whirl of a cantilevered turbofan. The effect of shaft torsional flexibility on the vibrations of rotating blades has not been considered in the previous studies. Okabe et al. [4] highlighted the necessity for modelling both blade bending and shaft torsional deformations in turbo-machinery. Al-Bedoor [5], based on multi-body dynamic approach, developed a coupled model for shaft-torsional and blade-bending vibrations in rotors. The model employed the finite element method to discretize the blade deformations. The study identified the non-linear interaction and the destabilizing effect that the blade and shaft could introduce to excite each other. Due to the difficulty encountered in quantifying the nature of non-linear coupling when the finite element method is used, a reduced order non-linear dynamic model for shaft-torsional and blade-bending vibrations which adopted the assumed modes method (AMM) for approximating blade deformations was reported by Al-Bedoor [6]. The simulation results showed that the torsional vibration of the rotor system works as an excitation to the blade.

The purpose of the present letter is to extract an equation from the general model developed in reference [6]. This equation describes the rotating blade vibration under the effect of shaft-torsional vibration. A transformation is employed that converts the

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differential equation into a form similar to the Mathieu equation which is simulated to show the dynamic behavior of the blade under the effect of torsional vibration excitation.

2. THE DYNAMIC MODEL

A schematic diagram of a disk–shaft–blade system driven by an electrical motor is shown in Figure 1. The disk is assumed to be rigid and the flexible blades are attached radially to the disk as shown. The beam is assumed to be inextensible and the Euler–Bernoulli beam theory was adopted. The model adopted the small deformation theory for both blade-bending and shaft-torsional deformations. The co-ordinate systems used in developing the model are shown in Figure 2. Wherein, XY is the inertial reference frame, $x^m y^m$ is a body co-ordinate system of the motor shaft, $x^d y^d$ is a body co-ordinate system of the disk and $x^b y^b$ is the blade co-ordinate system that is attached to the root of the blade such that $x^b$is always directed along the undeformed blade centerline.

The degrees of freedom are the rigid body rotation θ, the torsional deflection ψ and the blade modal deflection $\{\mathbf{q}\}$. By considering only the blade modal degree of freedom and under the assumptions of constant rotating speed $\dot{\theta} = \Omega$, and the square of the torsional deflection is small $\psi^2 \approx 0$, the following equation can be obtained:

$$(h + \psi q)\ddot{\psi} + \ddot{q} + 2\psi\dot{\psi}\Omega + (2\psi\dot{\psi} + 2\eta\omega_B)\dot{q} + (\omega_B^2 + C_1(\dot{\theta} + \dot{\psi})^2 + \dot{\psi}^2)q = 0, \tag{1}$$

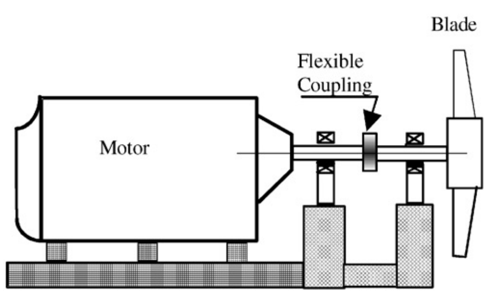

Figure 1. Schematic diagram of blade-disk-shaft system.

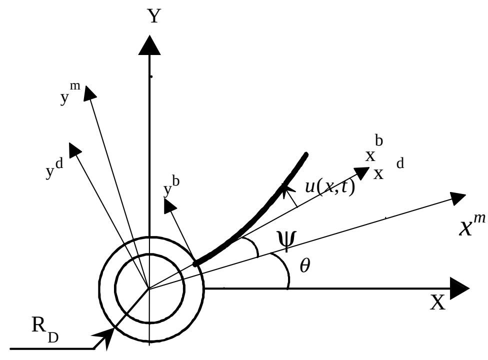

Figure 2. System deformed configuration and the coordinate systems.

where h and C_1 are constants related to the considered blade bending mode and ω_B is the blade natural frequency, [6]. The constant C_1 can be taken as unity as can be inferred from the numerical values of reference [6], with out loss of generality. The first term in equation (1) represents part of the dynamic coupling between the shaft torsional degree of freedom and the blade bending modes, which will work as forcing input to the system and will be taken to the right-hand side of equation (1). The second term is the blade modal acceleration with the coefficient of unity due to normalization process. The third is another term, which is a function of the torsional degree of freedom parameters and will be taken to the right-hand side of equation (1) as a forcing term. The blade damping terms are shown in the fourth term of equation (1), wherein, the structural damping is represented by η and the term $2\psi\dot{\psi}$ is the effect of torsional vibration. Finally, the fifth term of equation (1) is the generalized modal stiffness in which the effect of stiffening due to rotation and torsional vibration is apparent.

Now, assuming that the rotor torsional deflection can be represented by $\psi = \varepsilon \sin \omega t$, where ε is a small parameter that indicates the small order of torsional vibration and ω is the torsional excitation frequency. Substituting for ψ and $\dot{\psi}$ into equation (1) and arranging, one can find that the equation of motion takes the following form

$$\ddot{q} + P_1(t)\dot{q} + P_2(t)q = f(t), \tag{2}$$

where

$$\begin{aligned} P_1(t) &= \varepsilon^2\omega \sin(2\omega t) + 2\eta\omega_B, \\ P_2(t) &= \omega_B^2 + \Omega^2 + \tfrac{1}{2}\varepsilon^2\omega^2 + 2\varepsilon\omega\Omega \cos \omega t + \tfrac{3}{2}\varepsilon^2\omega^2 \cos 2\omega t, \\ f(t) &= \varepsilon h\omega^2 \sin 2\omega t - \varepsilon^2\omega\Omega \sin 2\omega t. \end{aligned}$$

Now, using the transformation given by Nayfeh [7],

$$q = x \exp\left(-\tfrac{1}{2}\int P_1(t)\,\mathrm{d}t\right)$$

and considering the homogenous part of equation (2) (i.e., droping the forcing term $f(t)$), the system can be expressed in the following form:

$$\dot{x} + P(t)x = 0, \tag{3}$$

where

$$P(t) = P_2 - \tfrac{1}{4}P_1^2 - \tfrac{1}{2}\dot{P}_1.$$

Upon substituting the full expression of $P(t)$ into equation (3), collectting the terms and defining a set of coefficients, the governing equation becomes

$$\begin{aligned} &\ddot{x} + (\alpha_1 + \beta_1\cos(\omega t) + \beta_2\cos(2\omega t) \\ &+ \beta_3\cos(4\omega t) + \lambda_1\sin(2\omega t)x = 0, \end{aligned} \tag{4}$$

where the coefficients are

$$\alpha_1 = \omega_B^2 + \Omega^2 + \tfrac{3}{8}\varepsilon^2\omega^2 - \eta^2\omega_B^2, \tag{5}$$

$$\beta_1 = 2\varepsilon\Omega\omega, \tag{6}$$

$$\beta_2 = \tfrac{1}{2}\varepsilon^2\omega^2, \tag{7}$$

$$\beta_3 = \tfrac{1}{8}\varepsilon^4\omega^2, \tag{8}$$

$$\lambda_1 = -\varepsilon^2\omega\omega_B\eta. \tag{9}$$

Equation (4) is a second order differential equation with harmonic coefficients. The frequency of the coefficients terms is the torsional vibration frequency with even multiples. The obtained equation (4) is a standard Mathieu–Hill equation with multiple harmonic coefficients that can be helpful for more studies on the stability of rotating blades under the effect of shaft torsional vibration excitations. It is worth mentioning that one can think of dropping the terms multiplied by higher orders of ϵ, to have similar treatment as dropping ψ^2. However, when the coefficients the ϵ and its higher orders are multiplied mainly by ω^2 which renders the total contribution not small and thus cannot be dropped.

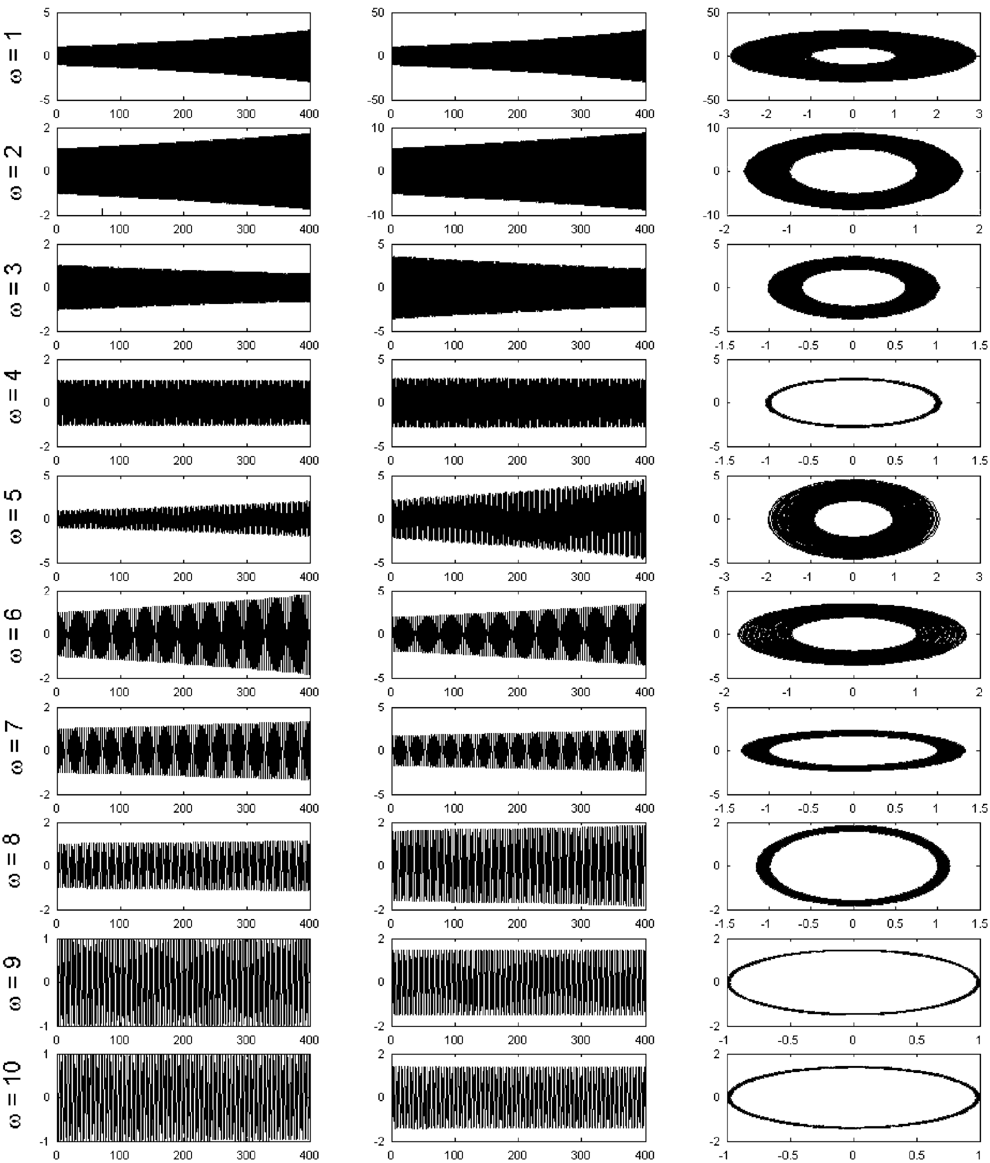

Figure 3. Blade vibration for t torsional excitation frequencies ($\omega = 1-10$ Rad/s).

3. NUMERICAL SIMULATION AND DISCUSSION

Equation (4) is integrated in time using the MATLAB package. The blade first mode natural frequency is chosen as $\omega_B = 10\,\mathrm{rad/s}$ and the torsional vibration excitation frequency is assumed to be equal to the running speed ($\omega = \Omega$); the well-known 1X excitation. The small parameter ϵ is taken as 0·1 for all simulations, however, other values were tried. In order to show the system basic response, no damping is added and only the undamped system is considered. The simulation results are shown in Figures 3 and 4 for the range of torsional excitation frequency of $\omega = 1-20\,\mathrm{rad/s}$ in steps of 1 rad/s. The simulation output is given as modal deflection, modal velocity and the phase plane plots.

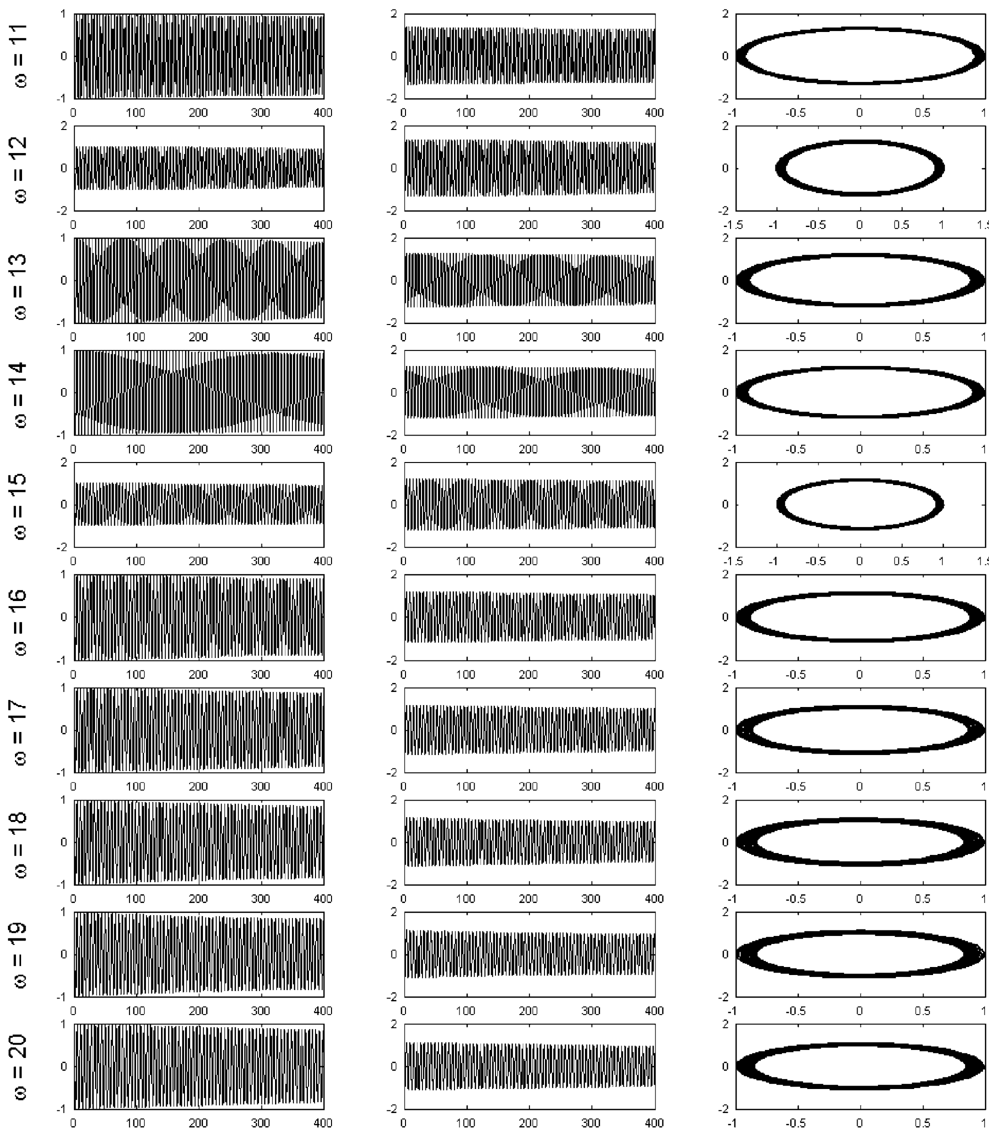

Figure 4. Blade vibration for t torsional excitation frequencies ($\omega = 11 - 20$ Rad/s).

For torsional vibration excitation frequency range $\omega = 1-3$ rad/s, the blade vibration shows growing amplitude, which can be considered as an unstable behavior. For torsional excitation frequencies $\omega = 3$ and 4 rad/s, the blade modal vibration amplitude is decreasing and constant respectively. For frequencies $\omega = 5, 6, 7$ and 8 rad/s, the blade modal vibration amplitude is increasing, but at a reduced rate when going from 5 to 6 to 7 and to 8, as shown, until it becomes constant sustained for $\omega = 9-20$ rad/s. This shows that the critical regions of torsional excitation frequencies exist and can lead to unstable blade (growing) vibration. The critical regions occur in torsional excitation frequencies that are lower than the blade bending natural frequency. This preliminary result means that the danger of unstable blade bending vibration due to shaft torsional excitation exists when exposing the system to torsional vibration excitation frequencies that are lower than the blade natural frequency.

4. CONCLUSIONS

A non-linear dynamic model for blade bending vibration under the effect of shaft torsional flexibility is considered in this paper. The model is reduced to an ordinary differential equation, which is transformed to an equation similar to Mathieu equation with multiples of harmonic coefficients. The coefficients of the obtained equation contain the system parameters such as the rotating speed, the shaft torsional vibration excitation frequency and the blade bending natural frequency. The differential equation is simulated and the results showed regions of unstable blade vibrations when the torsional excitation frequency is lower than the blade bending natural frequency. Further analytical and experimental studies in this direction are recommended in order to extract rigid conclusions.

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